# A Correction to
# Affine transformations, polynomials, and proportionality

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We would like to thank several careful readers of our article [**1**] for pointing out to us that Theorem 2 therein is incorrect as stated. The other results in the paper, concerning a variety of proportionality problems for parabolas and cubic curves, are not affected by this error. The correct statement of the theorem should be as follows.

**Theorem 2′.** Let $n \geq 4$ and let $p(x) = \sum_{j=0}^{n} \alpha_j x^j$ and $q(x) = \sum_{j=0}^{n} \beta_j x^j$ be polynomials such that $\alpha_n = \beta_n = 1$, $\alpha_{n-1} = \beta_{n-1} = 0$, and $\alpha_0 = \beta_0 = 0$. Then $p$ and $q$ are affinely equivalent if, and only if, there exists a non-zero number $a$ such that $\beta_j = a^{n-j}\alpha_j$ for all $j \neq 1$. In this case, the affine transformation $T(x,\, y) = (ax,\, (a\beta_1 - a^n\alpha_1)x + a^n y)$ implements the equivalence.

*Proof.* As discussed in [**1**], the restrictions on the coefficients of $p$ and $q$ involve no loss of generality as every polynomial of degree $n$ is affinely equivalent to one such as this.

Assume that $p$ and $q$ are affinely equivalent, implemented by the affine transformation $T(x,\, y) = (ax+by+e,\, cx+dy+f)$ with $ad-bc \neq 0$. Thus, $q(ax+bp(x)+e) = cx+dp(x)+f$ for all $x$. Comparisons of the coefficients of the polynomials $q(ax+bp(x)+e)$ and $cx+dp(x)+f$ show that $b = e = f = 0$ and that the non-zero number $a$ satisfies $\beta_j = a^{n-j}\alpha_j$ for all $j \neq 1$. Moreover, $c = a\beta_1 - a^n\alpha_1$ and $d = a^n$.

Conversely, if there is a non-zero number $a$ for which $\beta_j = a^{n-j}\alpha_j$ whenever $j \neq 1$, then it is straightforward to verify that the affine transformation $T(x,\, y) = (ax,\, (a\beta_1 - a^n\alpha_1)x + a^n y)$ satisfies the condition $T(x,\, p(x)) = (ax,\, q(ax))$ for all $x$. That is, $T$ implements an affine equivalence between $p$ and $q$. □

For $n = 4$, this result implies that there are exactly three affine equivalence classes of quartic polynomials, represented by $x^4$, $x^4 + x^2$, and $x^4 - x^2$. For $n \geq 5$, however, there are infinitely many affine equivalence classes. For instance, the quintics $x^5 + \alpha x^3 + x^2$ and $x^5 + \beta x^3 + x^2$ are affinely equivalent if, and only if, $\alpha = \beta$.

## REFERENCE

1. Timothy G. Feeman and Osvaldo Marrero, Affine transformations, polynomials, and proportionality, *American Math. Monthly* **108** (2001) 972–975.

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